Optimal Randomized Complete Visibility on a Grid by Asynchronous Robots with Lights

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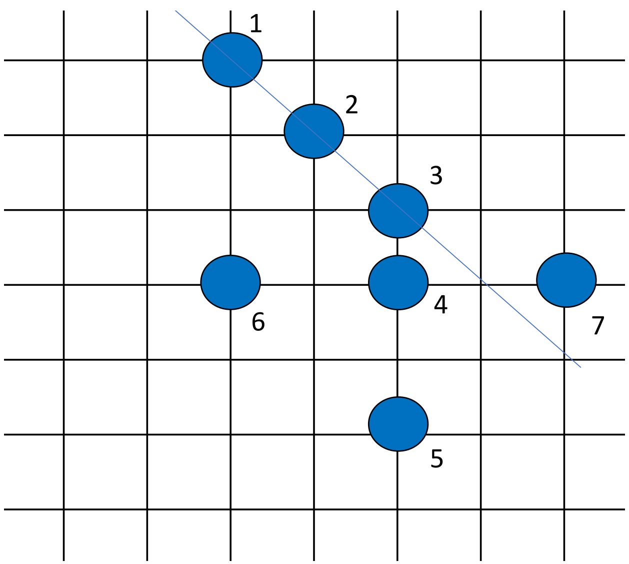
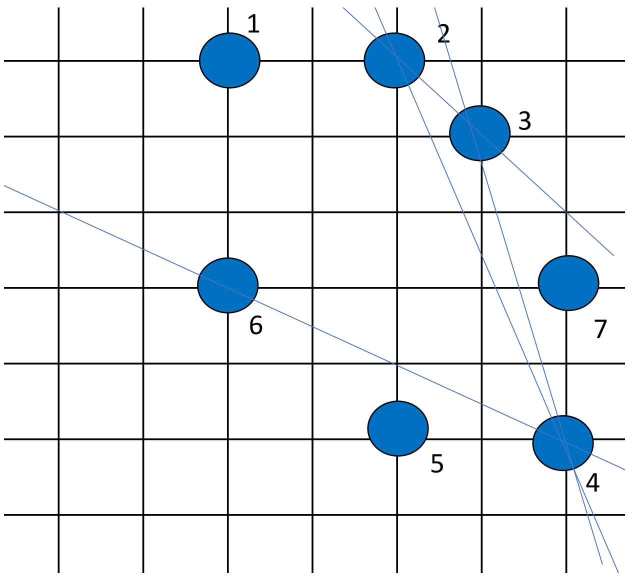
The Complete Visibility Problem on a Grid

Input: N robots placed initially arbitrarily on distinct nodes of a grid

Output: Each robot is on a distinct node of the grid and it sees all $N-1$ others



Input of 7 robots: 1 can't see 3, 3 can't see 1,5, and 6(7) can't see 7(6)



Output: each robot sees everyone else

Robot and Grid Model

Point Robots with Lights:

- Anonymous, autonomous, indistinguishable, disoriented
- Obstructed visibility for collinear robots
- Equipped with lights that can display a color at a time from a fixed set; the light colors are persistent

Grid:

- Grid embedded in the Euclidean plane
- Grid nodes have no IDs and edges have no labels
- Nodes do not have memory
- Unbounded size
- Applications and use in real-life robotic systems

Performance model

Asynchronous - all robots perform their cycles (below) at arbitrary times

Epoch – the time interval for every robot executing its cycle at least once

Runtime – the number of epochs

Area – the grid size occupied by the solution

A cycle for a robot:

Look: observe positions and colors of all visible robots

Compute: compute destination node to move and color for the light; destination node is one of the four neighbor nodes

Move: change the color and move to the destination node

Contributions

Result	Time	Area	No. of Colors	Remarks
Lower bound	$\Omega(N)$	$\Omega(N^2)$	--	--
Upper bound	$O(\max\{D, N\})$	$O(N^2)$	17	deterministic
Upper bound	$O(\max\{D, N\})$	$O(N^2)$	50	randomized

- D- diameter of the input configuration
- Our upper bounds are optimal on time when $D = O(N)$
- Our upper bounds are always optimal on area
- Deterministic/randomized depends on leader election requirement

Previous Result

Adhikary et al.	$--$	$--$	11	deterministic
This paper	$\Omega(\max\{DN, N^2\})$	$\Omega(N^2)$	--	deterministic

Improvement on time at least $O(N)$ factor, keeping number of colors $O(1)$

Techniques

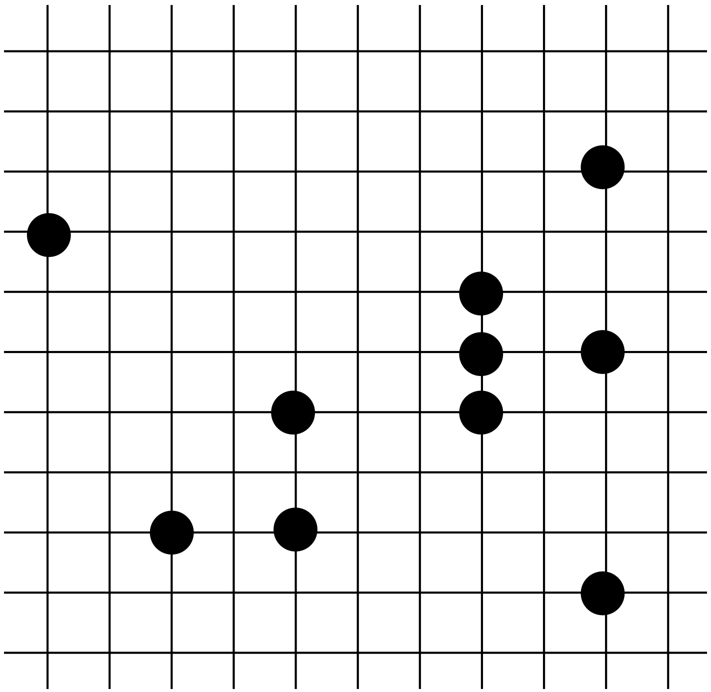
Lower bound – pigeon-hole argument of no three robots can be on a horizontal/vertical line of grid for complete visibility

Upper bound – 3 Stages (Stage 1 – 3)

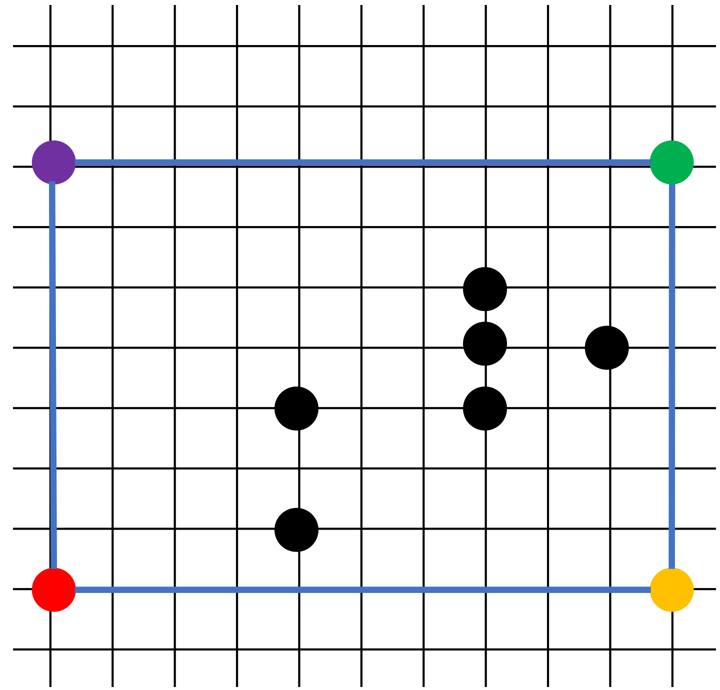
- Stage 1 – elect two leaders if needed
- Stage 2 – move robots to position themselves on an axis-aligned (horizontal/vertical) line
- Stage 3 - move robots from the line to the grid nodes satisfying complete visibility

Each stage runs for $O(\max\{D, N\})$ epochs

Stage 1



Input of 10 robots



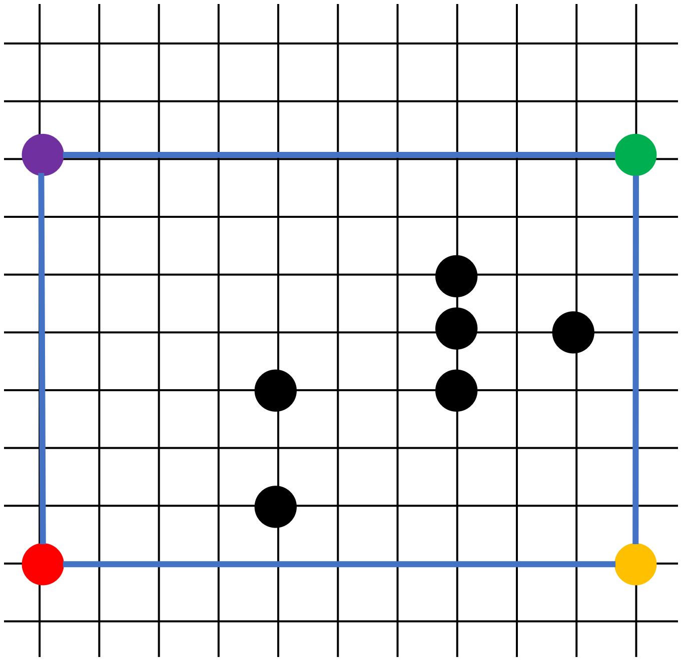
After Stage 1

2-step process:

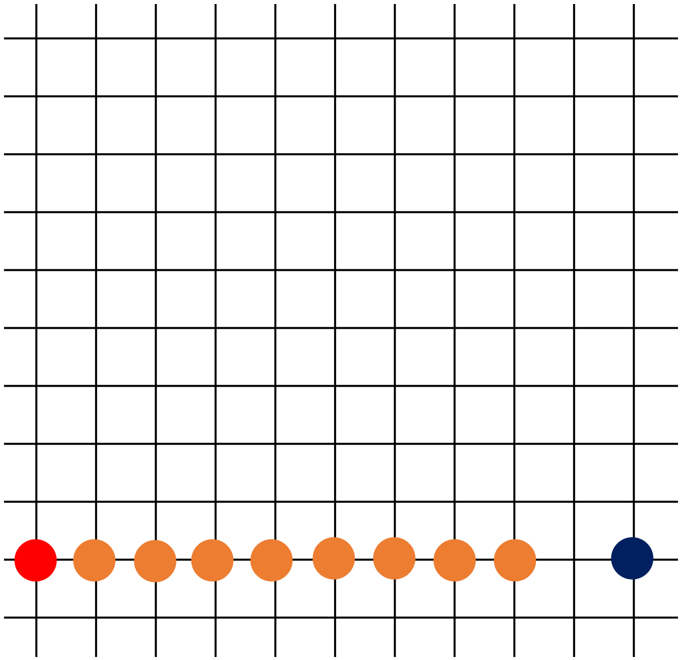
Step 1: Arrange robots on a four-corner axis-aligned rectangle configuration (the right figure); $N - 4$ robots are in its interior

Step 2: Elect two leaders among four robots on the rectangle; red and yellow; yellow and green; green and purple; or purple and red

Stage 2



After Stage 1



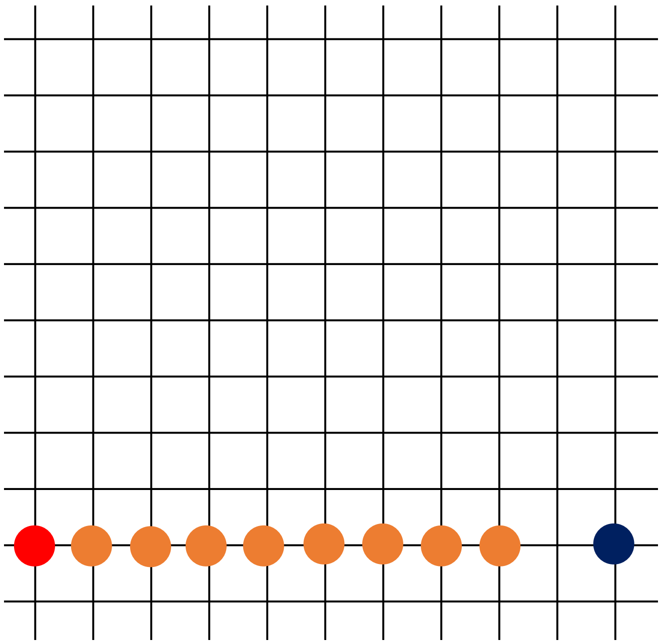
After Stage 2

-Suppose red and yellow were elected first and second leader in Stage 1

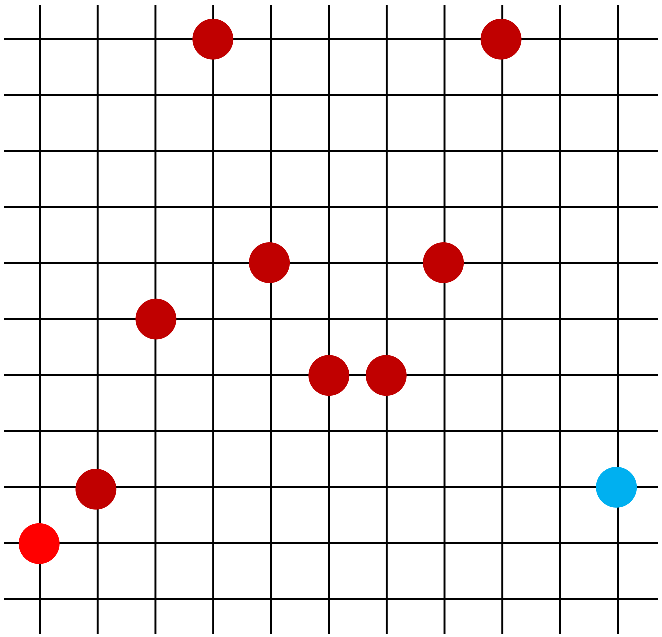
-Move all robots to the red-yellow line on consecutive positions (the right figure)

-If N is not prime, move one robot to the first prime $N' > N$ distance from red (black robot on the right figure)

Stage 3



After Stage 2



After Stage 3 (a complete visibility configuration)

- -Move black robot one hop up and color different
- -Red robot stays wherever it was after Stage 2
- -For each other robot, if it is at distance i from Red then move vertically up distance $i^2 \, mod \, N'$ and assume some different color
- -From Roth's theorem [19], complete visibility is guaranteed

Thank You!

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Resources:

http://www.ece.lsu.edu/vaidy/IPDPS-20-Resources/